Computing Pi from Fibonacci Numbers

11505 [2010, 458]. *Proposed by Bruce Burdick, Roger Williams University, Bristol, RI.* Define $\{a_n\}$ to be the periodic sequence given by $a_1 = a_3 = 1$, $a_2 = 2$, $a_4 = a_6 = -1$, $a_5 = -2$, and $a_n = a_{n-6}$ for $n \geq 7$. Let $\{F_n\}$ be the Fibonacci sequence with $F_1 = F_2 = 1$. Show that

$$\sum_{k=1}^{\infty} \frac{a_k F_k F_{2k-1}}{2k-1} \sum_{n=0}^{\infty} \frac{(-1)^{kn}}{F_{kn+2k-1} F_{kn+3k-1}} = \frac{\pi}{4}.$$